A continuous time stochastic model for biological neural nets

Leonardo Nagami Coregliano
IME – Universidade de São Paulo
This work was partially supported by CAPES and FAPESP grant no. 2013/23720-9

Our main goals

- To model mathematically a biological neural net as a continuous time stochastic process (to extend a model by Galves & Löcherbach (2013) from discrete time to continuous time);
  - Has been done by Duarte & Ost (2014).
- To study this model.
  - Does the system "die"?
  - How does the system "die"?

Our approach (our subgoal)

- To produce a model that can be easily simulated.
  - Stochastic differential equation (Duarte & Ost) approach does not work;
  - Adapt the discrete time model to continuous time by adapting one of its simulation algorithms to continuous time;
    - Downside: our model is not as general;
    - Upside: model's existence comes for free.

The model

- Finite set of neurons I;
- At time t, each neuron i has a membrane potential $U_t(i)$;
- Each neuron i has a potential probability function φ_i: the probability of i firing between time t and $t + dt$ is $\varphi_i(U_t(i))\,dt$;
- Matrix of influences W: every time a neuron i fires, the potentials of all neurons are adjusted:
  - Neuron $j \neq i$: $U_t(j) \to U_t(j) + W_{i\to j}$
  - Neuron i: $U_t(i) \to 0$
- Each neuron i has a potential decay function V_i : Without neuron discharges, after s time has passed, the potential goes from u to $V_i(u, s)$.

The model must make sense…

- Mathematically:
  - Integrability conditions.
- Biologically:
  - Non-negative potentials $U_t(i)$;
  - Non-decreasing potential probability functions φ_i;
  - Decay function $V_i(u,s)$ non-decreasing in u and non-increasing in s.
- Philosophically:
  - Interruption conditions on $V_i(u,s)$.

The simulation algorithm

- Instead of computing whether a neuron fires or not, we compute the waiting time for it to fire;
  - Involves calculating the inverse of a cumulative distribution function.

Potential Time $t = 3$	Waiting time
3.3	2
7.2	1.5
4.9	5
0	∞
3.2	∞

The simulation algorithm

- Instead of computing whether a neuron fires or not, we compute the waiting time for it to fire;
  - Involves calculating the inverse of a cumulative distribution function.

Potential Time $t = 3$	Waiting time	Potential Time	Potential Time $t = 4.5$
3.3	2	0.825	1.825
7.2	1.5	1.8	0
4.9	5	1.225	2.225
0	∞	0	1
3.2	∞	0.8	1.8

Studying the model

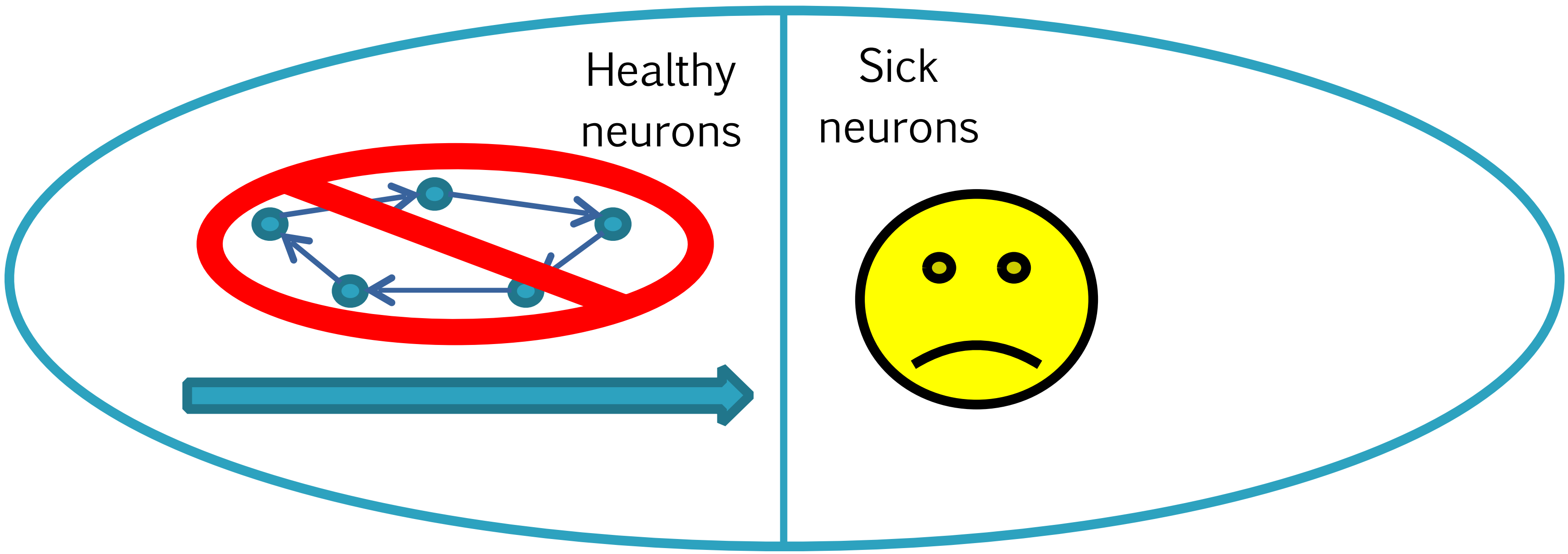
- System death: no neuron discharges after a certain time t.
- Question: does the system die in finite time?
  - Conjecture: it depends...
    - On the relation potential decay vs. probability potential;
    - On the influences between the neurons (both values and structure).
- Question: what is the distribution of the time of death (time of last discharge)?
  - Conjecture: it depends...
    - On the relation potential decay vs. probability potential;
    - On the influences between the neurons (both values and structure).

A theorem on system death

- Hypotheses:
  - Non-negative influences $W_{i \to j} \geq 0$;
  - Positive initial potentials $U_0(i) > 0$;
  - [...].
- Two types of neurons:
  - Healthy: if it has potential, it fires in finite time;
  - Sick: even if it has potential, the waiting time may be infinite.

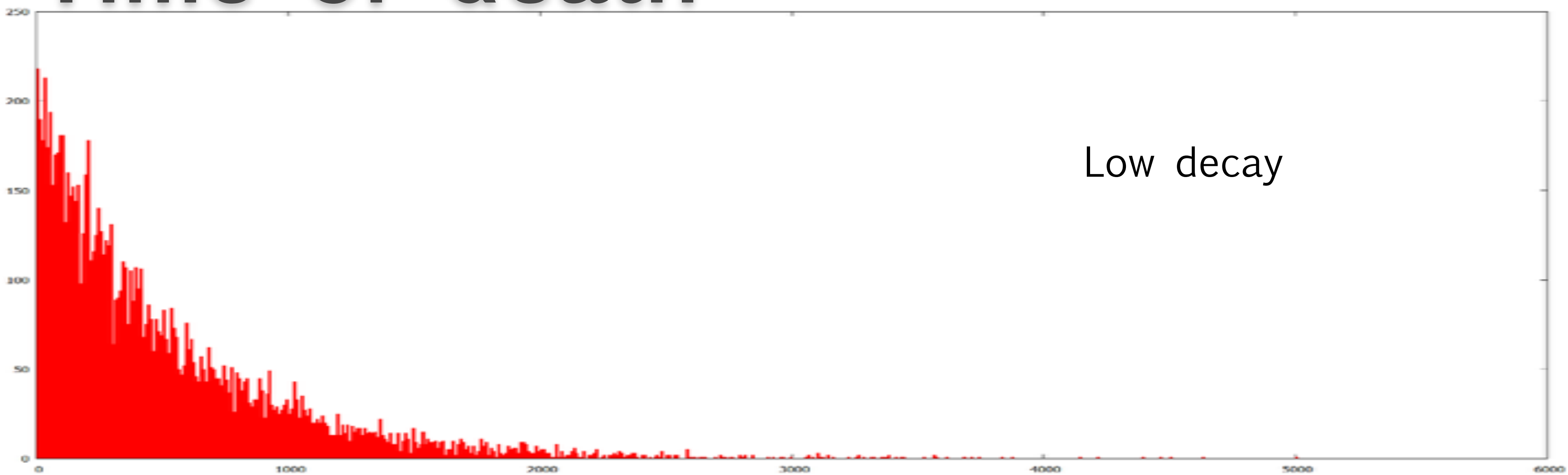
$$\forall u, \int_0^\infty \varphi_i(V_i(u,s))\, ds < \infty \Leftrightarrow \text{Sick}$$

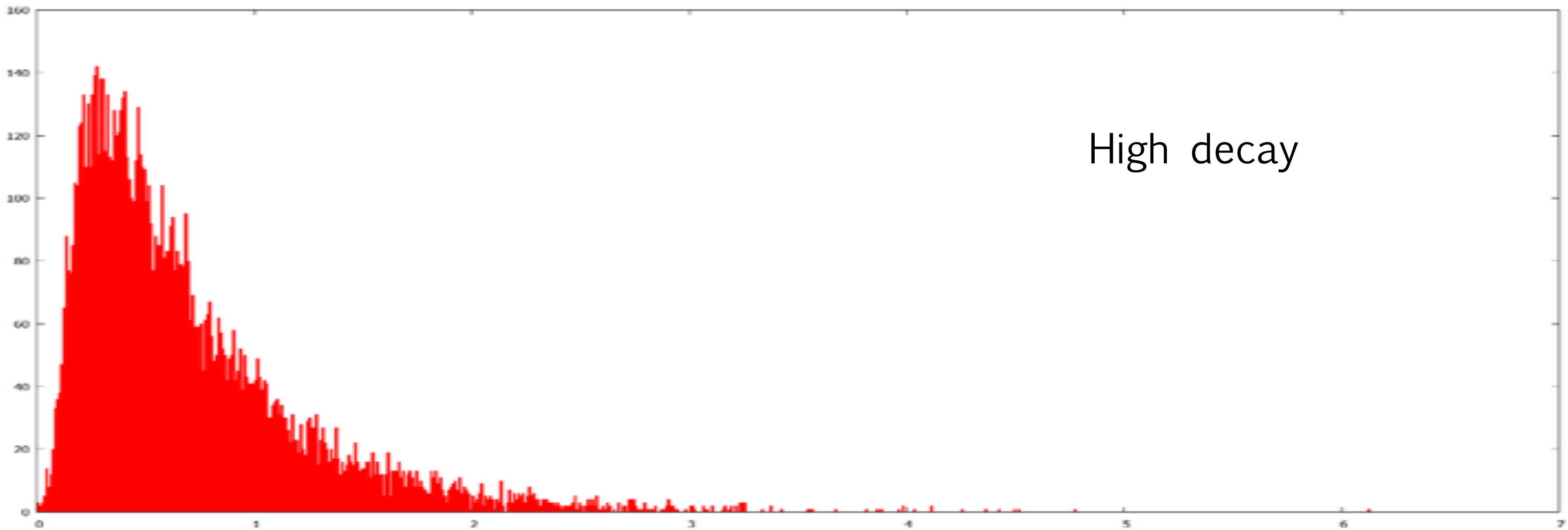
A theorem on system death



- The system dies in finite time with positive probability if and only if there is no cycle on the healthy neurons.
- Furthermore, if the system dies in finite time with positive probability, then it dies in finite time with probability one.

Time of death





Future directions

- Distribution of time of system death;
- What about negative influences $W_{i \to j}$?
- Distribution of potentials;
- Number of discharges until system death;
- Invariant probability measures in the case the system does not die.

Thank you!